Fostering Development of Quadrilateral Hierarchy in Geometer's Sketchpad

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Quadrilaterals and their interrelationships are important part of school mathematics. The current literature about quadrilaterals piles up within three main categories of research focusing on: (1) Students' understanding of and ability to identify quadrilaterals that cut across school mathematics and their interrelationships (e.g., Fujita & Jones, 2007); (2) van Hiele (VH) levels of geometric thought and how students fall into a variety of levels for different geometry topics (Jones, 2000); (3) The impact of technology-supported environments on student achievement. However, these categories of research fall short in explaining how to foster student thinking from one VH level to another. Therefore, this study aimed to investigate the core pieces of a technology-supported (Geometer's Sketchpad (GSP)) instructional sequence that help to foster a deep understanding of the hierarchy of certain quadrilaterals (square, rectangle, rhombus, parallelogram, trapezoid) as students shift from one VH level to another.

The current qualitative study benefitted from teaching experiment methodology in designing teaching sessions. Participants were two volunteered average fifth graders, one boy and one girl, chosen from among several students based on pre-interviews. We applied a technology-intensive curriculum (an adaptation of Battista's (2012) Shape Makers) to participants individually to investigate their progression from one VH level to another. All teaching sessions were videotaped, and all computer work was captured. The analyses included teaching sessions as well as pre-/post-interview analyses.

In designing the tasks, we considered participants' available understandings and searched for ways to help them use those understandings to conceptualize the unknown interrelationships among targeted quadrilaterals. In so doing we targeted participants' focusing on "figures" as opposed to "drawings" (Parzysz, 1988) as their thinking were fostered from one VH level to another. We targeted "inclusive definitions" of targeted quadrilaterals (Usiskin & Griffin, 2008).

We found that there are four main steps in helping students shift from considering quadrilaterals in isolation (VH1) to understanding them as inclusive of one another (VH2). An example of these steps for fostering an understanding of the interrelationship between squares and rectangles is given below.

(1) We first had students use the given curriculum in GSP to understand that a rectangle can make squares when dragged. Though, such work alone did not allow them to understand the interrelationships among squares and rectangles since their focus was on *drawings* and glaring properties of those *drawings* as opposed to *figures*.

(2) We then focused students' attention on limitations and flexibilities of those *drawings*. For example, a *limitation* of a square is it having four equal sides whereas there is no such limitation for rectangles (which is a *flexibility*). However, such an understanding about limitations and flexibilities in isolation for square and rectangle drawings did not help students understand the interrelationships.

(3) Then we helped students understand *how a limitation for a figure always implies a flexibility*

for another figure. For example, having four equal sides for a square (a limitation) always implies two pairs of oppositely equal sides (a flexibility); therefore, a square always has to be included within rectangle set. We asked participants to reflect on the idea that 'one limitation implies a flexibility' through certain probing questions. Such reflection helped students understand why a limitation implies a flexibility. However, this was not enough for understanding 'a square is always a rectangle'.

(4) We realized that the language of 'a square is a rectangle' used in classrooms and textbooks meant for participants that 'squares and rectangles are copies of each other, like twins.' We therefore changed our use of language from 'a square is a rectangle' to 'is a square included in rectangle family because of its properties?' Such a change in language gave students the opportunity to reflect on the fact that four-equal-side property of squares always implies opposite-pairs-of-equal-sides property of rectangles.

We found this sequence of four steps as a key in helping students' thinking progress from VH1 to VH2 for understanding the targeted quadrilaterals.

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